

# PID Position Control System Design for 2 DOF Propeller Pendulum Actuated by Four Motors

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## Abstract

In this paper, PID controller simulations for propeller pendulum systems actuated by single, double and four motors are investigated. PID controllers are designed for angular position control of a nonlinear propeller pendulum systems. Single propeller pendulum and double propeller pendulum systems have one degree of freedom. Also, four propeller pendulum system has two degrees of freedom. Propeller pendulum systems are actuated by the thrusts forces that are generated by four propellers at its free end and driven by PID controller. In this study, the mathematical models are obtained for single, double and four propeller pendulum systems, respectively. On the other hand, instead of the equations of motion of the system, the mechanical models are also used for control simulations. Results are compared with each other.

**Keywords:** PID control, Propeller, Pendulum, Mathematical model

## 1. Introduction

Propeller pendulum can be described as a motorized propeller at the end of the pendulum so that it can be lifted up or down by driving motor. Thrust force can be produced by driving propeller and pendulum can be stabilized at any desired angular position by using different control methods such as PID method, sliding modes method or fuzzy logic methods [1-4]. Propeller pendulum is a nonlinear system and can be controlled by classical control system after linearization.

The axis of rotation is fixed and the pendulum is actuated by four thrust force which is produced by propeller mounted at the free end of the pendulum. Four motorized propeller pendulum is capable of holding the pendulum any desired angular position on two planes by means of varying magnitude of thrust force. In literature, one or two motorized propeller pendulum systems exist. The angular position of the pendulum on a single plane is balanced against gravity for any desired position by means of brushed or brushless motor powered propeller [4]. The brushed or brushless motor can be controlled by PWM signals, so that pendulum system control behaviors such as stability, rise time overshoot etc. are adjusting.

Taskin proposed for angular position control based on fuzzy PID controller [4]. Mohammad Bagheri A. and Yaghoobi M. introduced PID control method for stabilizing pendulum based on

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the time response characteristics. PID method is an easy and simple tuning way to control pendulum [2].

In this paper, mathematical model and mechanic models of single, double and four propeller pendulum are obtained and control simulation results are compared to step input.

## 2. Mathematical model of propeller pendulum

Single propeller model is shown in Figure 1. In this model, the mass of pendulum rod and motor propeller couple are represented as  $m_1$  and  $m_2$ , respectively. As desired angular position  $\theta$ , proper propeller thrust force is applied.  $L$  and  $L_1$  represent the length of the pendulum, the distance between propeller rotation axis and pendulum rotation axis, respectively.  $J$  is the moment of the inertia of propeller pendulum system and for all different system, it is calculated. In simulations, APC 8x38SF and A2212/T12 (1400 kV) are selected as propeller - brushless motor couple. The relation between control signals and thrust force is predicted by using the relation between the control signal and RPM, and the relation between thrust force and RPM. These two relations are experimentally determined. Also, anti-torque of propeller system is neglected in all system. Model parameters value is given Table 1.

**Table 1.** Model Parameter Value

Parameter	SI Units	Value
$m_1$	kg	0.274
$m_2$ (motor propeller couple)	kg	0.058
$m_3$ (motor propeller couple)	kg	0.058
$m_4$ (motor propeller couple)	kg	0.058
$m_5$ (motor propeller couple)	kg	0.216
$L$	m	0.615
$L_1$	m	0.598
$L_2$	m	0.022
$g$ (gravity)	m/s <sup>2</sup>	9.80
$J_r$	kg-m <sup>2</sup>	0.0345

### 2.1. Mathematical model of single propeller pendulum

When viscous damping and disturbance torque of propeller are neglected, according to Newton's law and angular momentum, the equation of motion of single propeller pendulum is derived as follows:

$$M = \ddot{\theta}J \quad (1)$$

$$\ddot{\theta}J = TL_1 - m_1g \frac{L}{2} \sin \theta - m_2g L_1 \sin \theta - m_2g L_2 \cos \theta \quad (2)$$

$$\ddot{\theta}J + m_1g \frac{L}{2} \sin \theta + m_2g L_1 \sin \theta + m_2g L_2 \cos \theta = TL_1 \quad (3)$$

By considering  $\sin \theta \approx \theta$ ,  $\cos \theta \approx 1$  and also,  $L_2 \ll L$ , the modified equation of motion can be written as follows:

$$\ddot{\theta}J + m_1g \frac{L}{2} \theta + m_2g L_1 \theta = TL_1 \tag{4}$$

taking Laplace transform,

$$\theta(s)Js^2 + \theta(s)g[m_1 \frac{L}{2} + m_2L_1] = T(s)L_1 \tag{5}$$

The transfer function of the single propeller pendulum can be represented as follows:

$$\frac{\theta(s)}{T(s)} = \frac{L_1}{Js^2 + g[m_1 \frac{L}{2} + m_2L_1]} \tag{5}$$

where L: length of the pendulum, J: inertia moment, g: acceleration of gravity,  $m_1$ : weight of pendulum,  $m_2$  : weight of motor propeller couple,  $\theta$ : angular position,  $L_1$ : distance between the center of motor mass and rotation point, T: thrust force.

Schematic view of the single propeller pendulum system is given in Figure 1.

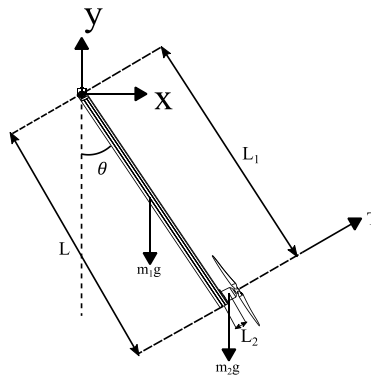


Figure 1. Schematic view of the single propeller pendulum system

PID control block diagram is given in Figure 2.

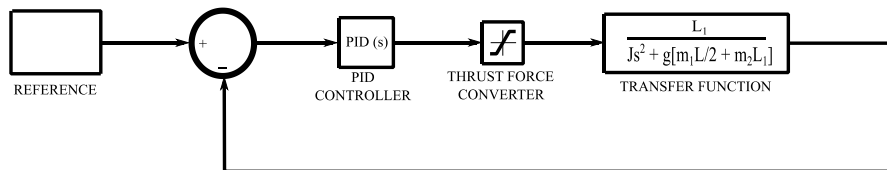


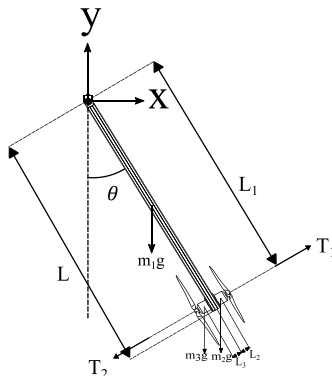
Figure 2. PID control block diagram of the single propeller pendulum system

**2.2. Mathematical model of dual propeller pendulum**

The transfer function of the double propeller pendulum can be written as follows:

$$\frac{\theta(s)}{T(s)} = \frac{L_1}{Js^2 + g[m_1 L/2 + (m_2 + m_3)L_1]} \tag{6}$$

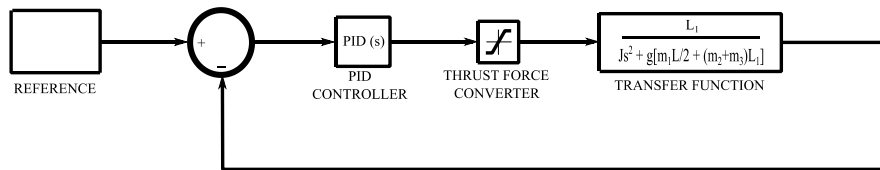
Schematic view of the double propeller pendulum system is given in Figure 3.



**Figure 3.** Schematic view of the double propeller pendulum system

where L: length of the pendulum, J: inertia moment, g: acceleration of gravity, m<sub>1</sub>:weight of pendulum, m<sub>2</sub>, and m<sub>3</sub>: weight of motor propeller couple, θ: angular position, L<sub>1</sub>: distance between the center of motor mass and rotation point, T<sub>1</sub> and T<sub>2</sub>: thrust force.

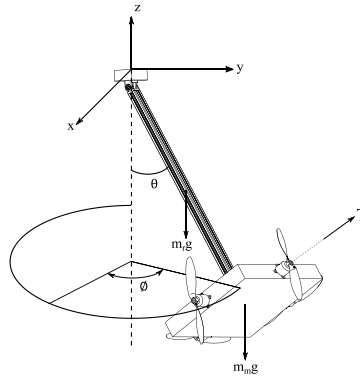
PID control block diagram of the double propeller pendulum system is given in Figure 4.



**Figure 4.** PID control block diagram of the double propeller pendulum system

**2.3. Mathematical model of four propeller pendulum**

Four propeller pendulum can be modeled as spherical rod pendulum. Four motors and propellers mass also can be considered as point mass due to attached to rod symmetrically. It is known that spherical rod pendulum has two degrees of freedom, so that, four propeller pendulum also has two degrees of freedom. θ and φ represent the position of the point mass. Schematic view of the spherical propeller pendulum system is given in Figure 5.



**Figure 5.** Schematic view of the spherical propeller pendulum system

The position of the point mass is represented by the vector  $r_0$ .

$$r_0 = \begin{bmatrix} L \sin \theta \cos \phi \\ L \sin \theta \sin \phi \\ -L \cos \theta \end{bmatrix} \quad (7)$$

The position of the rod mass is represented by the vector  $r_1$ .

$$r_1 = \frac{1}{2} \begin{bmatrix} L \sin \theta \cos \phi \\ L \sin \theta \sin \phi \\ -L \cos \theta \end{bmatrix} \quad (8)$$

Inertia of point mass with respect to O point is

$$J_m = m_m L^2 \quad (9)$$

where  $I_m$  : inertia of a point mass,  $m_m$ : point mass,  $L$ : length of the rod.

The inertia of rod with respect to O point is:

$$J_r = \frac{1}{3} m_r L^2 \quad (10)$$

where  $I_r$ : inertia of the rod,  $m_r$ : rod mass,  $L$ : length of the rod

Total inertia of the system is:

$$J_0 = \frac{1}{3} m_r L^2 + m_m L^2 \quad (11)$$

The kinetic energy:

$$T = \frac{1}{2} J_n |\dot{r}_n^2| \quad (12)$$

$$T = \frac{1}{2} J_0 [\dot{\theta}^2 + [1 - \cos \theta^2] \dot{\phi}^2] \quad (13)$$

The potential energy:

$$V = -m_n g L \cos \theta \quad (14)$$

$$V = -m_r g \cos \theta (L/2) - m_m g L \cos \theta \quad (15)$$

The Euler-Lagrangian equations:

$$L = T - V \quad (16)$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_n} \right) - \frac{\partial L}{\partial q_n} = Q_n \quad (17)$$

After applying Euler-Lagrangian equations and simplifying, mathematical model of four propeller pendulum is shown in the following,

$$\begin{bmatrix} J_0 & 0 \\ 0 & J_0 [1 - \cos \theta^2] \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} 0 & -J_0 \dot{\phi} \sin \theta \cos \theta \\ 2J_0 \dot{\theta} \sin \theta \cos \theta & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} (m_m + m_r/2)g \sin \theta \\ 0 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (18)$$

## 2.4. Mechanic Model

Using mechanic model is the way of saving time and effort for modeling and simulating mechanical systems which use the standard Newtonian dynamics of forces and torques. In the mechanic model, mechanical systems can be represented in a graphical way by connected block diagrams [5].

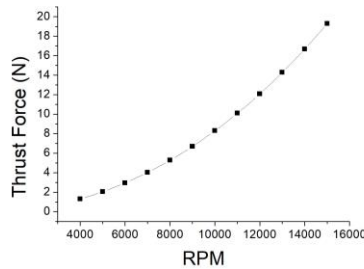
It is possible to validate the mathematical model by means of a mechanic model. Because of that, the validation of the mathematical models has been carried out through common control software. The CAD models firstly were designed in SolidWorks software. Afterthought, the cad models were imported to common control software. Block diagrams of mechanical models of a single propeller, double propeller, and four-propeller are given in Figure 6, 7 and 8, respectively.

The propeller pendulum systems can be effectively controlled without using the equation of motion in the mechanic model. To control the p of the pendulum, the thrust force which, is generated from propeller-motor couple, is used as an external force in simulations. External force values are generated by using thrust force equation based on the control signal. The modeling of thrust force is mentioned detailed in section 2.5.



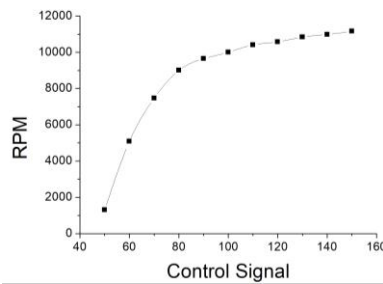
**2.5. Mathematical modeling for thrust force**

To obtain a mathematical model for the hobby type brushless motor is time-consuming and difficult due to determine motor parameters. Instead of using a mathematical model of brushless motor, the relation between the control signal and thrust force. Firstly, APC 8x38SF propeller performance data is obtained from the literature. Selig at al. experimentally obtain propeller thrust force data for different rpm values [6].



**Figure 9.** Thrust force vs. rpm diagram for APC 8x38SF propeller [6]

After that, the relationship between control signal and rpm for A2212/T12 (1400 kV) model brushless motor is obtained experimentally.

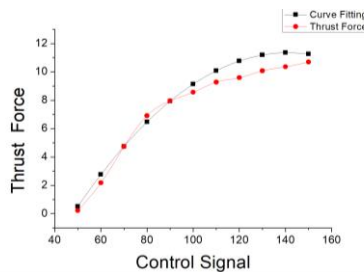


**Figure 11.** RPM vs. control signal diagram for A2212/T12 (1400 kV)

Using the obtained rpm values vs control signal and propeller performance data, second order equation is determined. Thrust force equation is given as following,

$$\text{Thrust force} = -0.0013x^2 + 0.3674x - 14.602 \tag{19}$$

In the simulation, a control signal is saturated between 0-150 and using thrust force equation, the required forces to control the systems is generated by the control signal.



**Figure 12.** Thrust force vs. control signal diagram



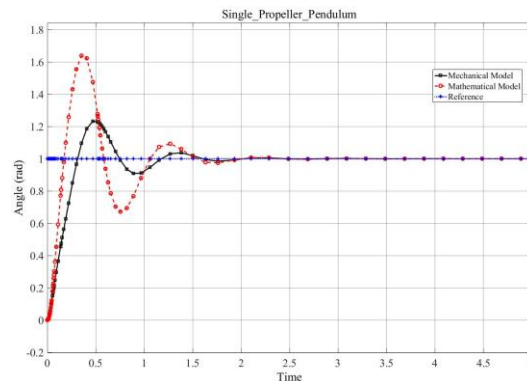
### 3. Results

A mathematical model of propeller systems is based on the parameter in table 1 and transfer functions for single, double pendulum are given equation 20 and 21, respectively.

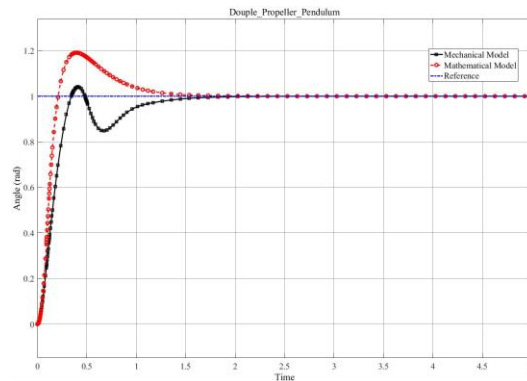
$$\frac{\theta(s)}{T(s)} = \frac{0.598}{0.0545s^2 + 1.116} \quad (20)$$

$$\frac{\theta(s)}{T(s)} = \frac{0.598}{0.076s^2 + 1.5} \quad (21)$$

Simulation durations are chosen 5 seconds for single and double propeller pendulum and 15 seconds for four propeller pendulum system. Position reference input is selected as step input. Classical PID control method is used to control the position of the pendulum systems. In the mathematical model, is limited to  $\pm 11$  N and external disturbances are neglected.



**Figure 12. Angular position of single propeller pendulum**



**Figure 13. Angular position of double propeller pendulum**

PID gains are found by trial-error methods for the mechanical models. After that, to compare the mechanical model's behavior with mathematical models, same PID gains are used in the mechanical model and mathematical model simulations. Single and double propeller pendulum simulation results are given in Figure 12 and Figure 13, respectively. Both models reach a stable position in 2 sn.

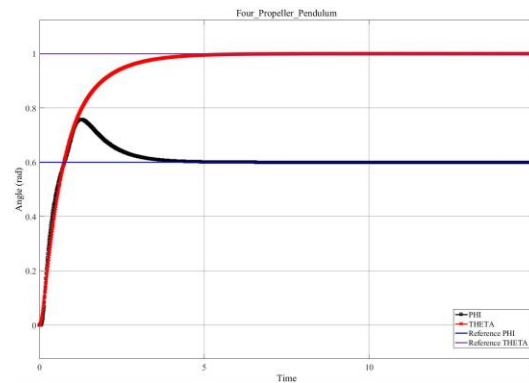


Figure 14. Angular positions of double propeller pendulum

Four propeller pendulum system reaches a stable position in 5 sn.

## Conclusions

PID control method is used to stabilize the position of three different propeller pendulum systems. The time responses of the pendulum systems were obtained for a step input for mechanical model control and mathematical model control methods. Using mechanical model is an easy and effective way to control the mechanical systems.

## Future works

The mathematical model of four propeller pendulum system is nonlinear because of that, for control simulation, these nonlinear system equations will be solved in future works and different control method is used for four propeller system and also experimental setup will be developed.

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